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Elementary Statistics
A Step by Step Approach
Eighth Edition

by
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SLIDES PREPARED
BY

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CHAPTER 8

Hypothesis Testing

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Objectives

- Understand the definitions used in hypothesis testing.
- State the null and alternative hypotheses.
- Find critical values for the *z* test.
- State the five steps used in hypothesis testing.
- Test means for samples using the *z* or t test.
- Test proportions for samples using the z test

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Introduction

- Statistical *hypothesis testing* is a decision-making process for evaluating claims about a population.
- In hypothesis testing, the researcher must
  1. define the population under study,
  2. state the particular hypotheses that will be investigated,
  3. give the significance level,
  4. select a sample from the population,
  5. collect the data,
  6. perform the calculations required for the statistical test,
  7. and reach a conclusion.

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Introduction

- Hypotheses concerning parameters such as means and proportions can be investigated.
- The *z* test and the *t* test are used for hypothesis testing concerning means.
- The *z* test is used for hypothesis testing concerning proportions.
- There are three methods used to test hypotheses:
  1. The traditional method.
  2. The confidence interval method.
  3. The *P*-value method.

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Introduction

- A *statistical hypothesis* is a conjecture about a population parameter which may or may not be true.
- There are two types of statistical hypotheses for each situation:
  1. The *null hypothesis*, symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
  2. The *alternative hypothesis*, symbolized by H_1, is a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

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State the null and alternative hypotheses for each conjecture.

- A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.

  $H_0: \mu = 8.6$ and $H_1: \mu > 8.6$ (Right-tailed test)

- An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.

  $H_0: \mu = 18$ and $H_1: \mu < 18$ (Left-tailed test)

- A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

  $H_0: \mu = 73$ and $H_1: \mu \neq 73$ (Two-tailed test)

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Introduction

- After stating the hypotheses, the researcher's next step is to design the study. The researcher selects the *correct statistical test*, chooses an appropriate *level of significance,* and formulates a plan for conducting the study.
- A *statistical test* uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.
- The numerical value obtained from a statistical test is called the *test value*.

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Introduction

Possible Outcomes of a Hypothesis Test

	H_0 True	**H_0 False**
Reject H_0	Error Type I	Correct Decision
Do not reject H_0	Correct Decision	Error Type II

- A *type I error* occurs if one rejects the null hypothesis when it is true.
- A *type II error* occurs if one does not reject the null hypothesis when it is false.

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Introduction

- The *level of significance* is the maximum probability of committing a type I error. This probability is symbolized by α; that is,

$$P\,(\text{type I error}) = \alpha$$

- The probability of a type II error is symbolized by β. That is,

$$P\,(\text{type II error}) = \beta$$

- In most hypothesis testing situations, β cannot easily be computed; however, α and β are related in that decreasing one increases the other.
- In a hypothesis testing situation, the researcher decides what level of significance to use.
- After a significance level is chosen, a *critical value* is selected from a table for the appropriate test.

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Introduction

- The *critical value* separates the critical region from the noncritical region. The symbol for critical value is C.V.
- The *critical* or *rejection region* is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.
- The *noncritical* or *nonrejection* region is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

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Types of Tests

- A *one-tailed* test indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean.
- A one-tailed test is either *right-tailed* or *left-tailed*, depending on the direction of the inequality of the alternative hypothesis.
- In a *two-tailed test*, the null hypothesis should be rejected when the test value is in either of the two critical regions.

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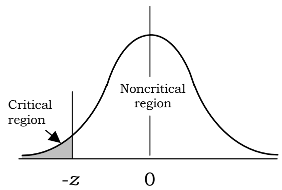
Introduction

Left-Tailed Test

$H_0: \mu = k$ $\quad \alpha = 0.10,\ C.V. = -1.28$

$H_1: \mu < k$ $\quad \alpha = 0.05,\ C.V. = -1.65$

$\alpha = 0.01,\ C.V. = -2.33$



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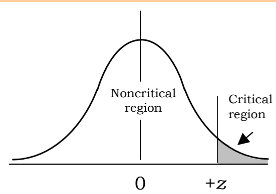
Introduction

Right-Tailed Test

$H_0: \mu = k$ $\quad \alpha = 0.10,\ C.V. = 1.28$

$H_1: \mu > k$ $\quad \alpha = 0.05,\ C.V. = 1.65$

$\alpha = 0.01,\ C.V. = 2.33$



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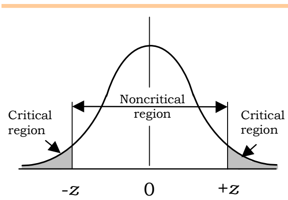
Introduction

Two-Tailed Test

$H_0: \mu = k$ $\quad \alpha = 0.10,\ C.V. = \pm 1.65$

$H_1: \mu \neq k$ $\quad \alpha = 0.05,\ C.V. = \mp 1.96$

$\alpha = 0.01,\ C.V. = \pm 2.58$



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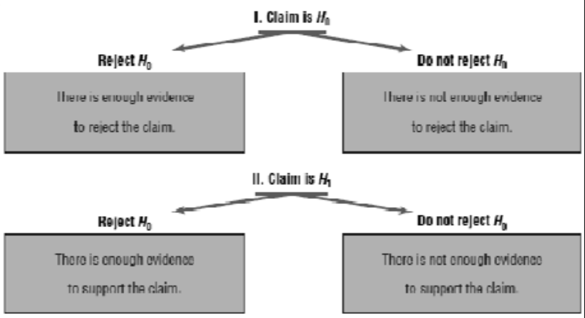

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Traditional Method

- Step 1: State the hypothesis, and identify the claim.
- Step 2: Find the critical value from the appropriate table.
- Step 3: Compute the test value.

$$\text{Test Vale} = \frac{\text{Observed Value} - \text{Expected Value}}{\text{Standard Error}}$$

- Step 4: Make the decision to reject or not reject the null hypothesis.
- Step 5: Summarize the results.

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z Test for a Mean

- The *z test* is a statistical test for the mean of a population.
- It can be used when σ is known and the population is normally distributed or $n \geq 30$.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$ = sample mean

μ = hypothesized population mean

σ = population standard deviation

n = sample size

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The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At , $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

$H_0: \mu = \$24,672$ and $H_1: \mu \neq \$24,672$

Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are -2.58 and 2.58.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{25,226 - 24,672}{3251/\sqrt{35}} = 1.01$$

Since the value of z is between -2.58 and 2.58, do not reject the null hypothesis. Thus, there is not enough evidence to support the claim that the average cost of rehabilitation is different from $24,672.

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A researcher reports that the average salary of assistant professors is more than $42000 per year. A sample of 30 assistant professors has a mean salary of $43260. At $\alpha = 0.05$, test the claim that assistant professors earn more than $42000 per year. The standard deviation of the population is $5230.

$H_0: \mu = \$42000$ and $H_1: \mu > \$42000$

Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is 1.65.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{43260 - 42000}{5230/\sqrt{30}} = 1.32$$

Since the test value, z=1.32, is less than the critical value,1.65, do not reject the null hypothesis. Thus, there is not enough evidence to support the claim that the average salary of assistant professors is more than $42000 per year.

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A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs in dollars. Is there enough evidence to support the researcher's claim at $\alpha = 0.10$. Assume σ=19.2.

60 70 75 55 80 55 50 40 80 70 50 95 120
90 75 85 80 60 65 80 85 85 45 75 60 110
90 90 60 95 85 45 90 70 70 110

$H_0: \mu = \$80$ and $H_1: \mu < \$80$

Since $\alpha = 0.10$ and the test is a left-tailed test, the critical value is -1.28.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{75 - 80}{19.2/\sqrt{36}} = -1.56$$

Since z=-1.56 is less than -1.28, reject the null hypothesis. Thus, there is enough evidence to support the claim that the average cost of men's athletic shoes is less than $80.

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t Test for a Mean

- The *t test* is a statistical test for the mean of a population.
- It can be used when σ is unknown and the population is normally distributed or $n \geq 30$.

$$z = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

- The degrees of freedom are d.f. = *n*–1.

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An educator claims that the average salary of substitute teachers in schools in his city is less than \$60 per day. A random sample of eight schools is selected, and the daily salaries in dollars are shown. Is there enough evidence to support the educator's claim at α=0.10?

60 56 60 55 70 55 60 55

$$H_0: \mu = \$60 \quad \text{and} \quad H_1: \mu < \$60$$

Since $\alpha = 0.10$ and the test is a left-tailed test, the critical values is -1.415.

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{58.88 - 60}{5.08/\sqrt{8}} = -0.624$$

Since the value of t=-0.624 > -1.415, do not reject the null hypothesis. Thus, there is not enough evidence to support the claim that the average salary of substitute teachers in educator's city is less than \$60 per day.

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A medical investigation claims that the average number of infections per week at a hospital is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Assuming the number of weekly infections is normally distributed, is there enough evidence to reject the investigator's claim $\alpha = 0.05$?

$$H_0: \mu = 16.3 \quad \text{and} \quad H_1: \mu \neq 16.3$$

Since $\alpha = 0.05$ and the test is a two-tailed test, the critical values are -1.96 and 1.96.

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Since the value of t=2.46 > 1.96, reject the null hypothesis. Thus, there is enough evidence to reject the claim that the average number of infections is 16.3.

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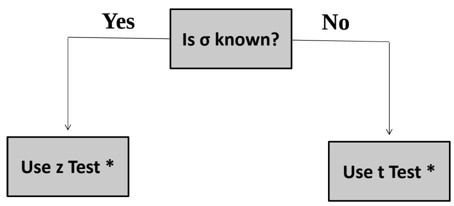
Using the z or t Test for a Mean



*** If n<30, the variable must be normally distributed**

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z Test for a Proportion

- A hypothesis test involving a population proportion can be considered as a binomial experiment.
- Since a normal distribution can be used to approximate the binomial distribution when *np≥5* and *nq≥5*, the standard normal distribution can be used to test hypotheses for proportions.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

$$\text{where} \quad \hat{p} = \frac{X}{n}$$

$$p = \text{population proportion}$$

$$n = \text{sample size}$$

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A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, he selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

$$H_0: p = 0.4 \quad \text{and} \quad H_1: p \neq 0.4$$

Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are -2.58 and 2.58. Note that *np>5* and *nq>5*.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.37 - 0.40}{\sqrt{0.4 \times 0.6/100}} = -0.612$$

Since the value of 2.58 > z=-0.612 > -2.58, do not reject the null hypothesis. Thus, there is not enough evidence to reject the claim that 40% of the telephone company's customers have call waiting.

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An attorney claims that more than 25% of all lawyers advertise. A sample of 200 lawyers in a certain city showed that 63 had used some form of advertising. At $\alpha = 0.05$, is there enough evidence to support the attorney's claim?

$$H_0: p = 0.25 \quad \text{and} \quad H_1: p > 0.25$$

Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is 1.65. Note that *np>5* and *nq>5*.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.315 - 0.25}{\sqrt{0.25 \times 0.75/200}} = 2.12$$

Since the value of z=2.12 > 1.65, reject the null hypothesis. Thus, there is enough evidence to support the attorney's claim that more than 25% of the lawyers use some form of advertising.

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Confidence Interval Method

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is not rejected, the confidence interval computed using the same level of significance will contain the hypothesized mean.
- Likewise, when the null hypothesis is rejected, the confidence interval computed using the same level of significance will not contain the hypothesized mean.

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The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

$$H_0: \mu = \$24,672 \quad \text{and} \quad H_1: \mu \neq \$24,672$$

Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are -2.58 and 2.58.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{25,226 - 24,672}{3251/\sqrt{35}} = 1.01$$

Since the value of z is between -2.58 and 2.58, do not reject the null hypothesis. The 99% confidence interval for the mean is given by

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \quad \Rightarrow \quad 23823.94 < \mu < 26628.06$$

Note that the 99% confidence interval contains the hypothesized value $24672.

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P-Value Method

- The *P-value* (or probability value) is the probability of getting a sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.
- Alternatively, the *P-value* is the actual area under the standard normal distribution curve (or other curve depending on what statistical test is being used) representing the probability of a particular sample statistic occurring if the null hypothesis is true.

- If $P-value \le \alpha$, reject the null hypothesis.
- If $P-value > \alpha$, fail to reject the null hypothesis.

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A researcher claims that the average cost of men's athletic shoes is less than \$80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at $\alpha = 0.10$?

60	70	75	55	80	55
50	40	80	70	50	95
120	90	75	85	80	60
110	65	80	85	85	45
75	60	90	90	60	95
110	85	45	90	70	70

H_0: $\mu = \$80$ and H_1: $\mu < \$80$ (claim)

Find the critical value. Since α 0.10 and the test is a left-tailed test, the critical value is -1.28.

$$z = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{75 - 80}{19.2/\sqrt{36}} = 1.56$$

1. Enter the data from Example 8-4 into column A of a new worksheet.
2. Select **MegaStat>Hypothesis Tests>Mean vs. Hypothesized Value.**
3. Select data input and enter the range of data A1:A36.
4. Type 80 for the Hypothesized mean and select the "less than" Alternative.
5. Select z test and click [OK].

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An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than \$60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at α 0.10?

60 56 60 55 70 55 60 55

H_0: $\mu = \$60$ and H_1: $\mu < \$60$ (claim).

At α 0.10 and d.f. 7, the critical value is -1.415.

To compute the test value, the mean and standard deviation must be found. Using either the formulas in Chapter 3 or your calculator, $\overline{X} - \$58.88$, and $s = 5.08$, you find

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{58.88 - 60}{5.08/\sqrt{8}} = -0.624$$

Do not reject the null hypothesis since -0.624 falls in the noncritical region.

1. Enter the data from Example 8-13 into column A of a new worksheet.
2. Select **MegaStat>Hypothesis Tests>Mean vs. Hypothesized Value.**
3. Select data input and enter the range of data A1:A8.
4. Type 60 for the Hypothesized mean, and select the "less than" Alternative.
5. Select t test and click [OK].

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A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, he selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

$H_0 : p = 0.4$ and $H_1 : p \neq 0.4$

1. From the toolbar, select Add-Ins,
   MegaStat>Hypothesis Tests>Proportion vs. Hypothesized Value.
2. Type 0.37 for the Observed proportion, p.
3. Type 0.40 for the Hypothesized proportion, p.
4. Type 100 for the sample size, n.
5. Select the "not equal" Alternative.
6. Click [OK]

Hypothesis test for proportion vs hypothesized value

Observed	Hypothesized	
0.37	0.4	p (as decimal)
37/100	40/100	p (as fraction)
37.	40.	X
100	100	n
	0.049	std. error
	-0.61	z
	.5403	p-value (two-tailed)

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Summary

- A statistical hypothesis is a conjecture about a population.
- There are two types of statistical hypotheses: the null hypothesis states that there is no difference, and the alternative hypothesis specifies a difference.
- Researchers compute a test value from the sample data in order to decide whether the null hypothesis should or should not be rejected.
- Statistical tests can be one-tailed or two-tailed, depending on the hypotheses.
- The null hypothesis is rejected when the difference between the population parameter and the sample statistic is said to be significant.

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Summary

- The difference is significant when the test value falls in the critical region of the distribution.
- The critical region is determined by α, the level of significance of the test.
- The significance level of a test is the probability of committing a type I error.
- A type I error occurs when the null hypothesis is rejected when it is true.
- The type II error can occur when the null hypothesis is not rejected when it is false.

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